# MATH 402 Homework 1

**Due Friday, September 8, 2017**

**Exercise 1.**

a. *[5 points]* Write down the definition of a group (see page 24 of the book).

b. *[10 points]* Let $G$ be the set of invertible functions from $\mathbb{R}$ to $\mathbb{R}$: an element of $G$ is a function $f : \mathbb{R} \to \mathbb{R}$ such that there exists another function $g : \mathbb{R} \to \mathbb{R}$ such that $f \circ g = g \circ f = \mathrm{Id}_{\mathbb{R}}$ (that is, such that for any $x \in \mathbb{R}, f(g(x)) = g(f(x)) = x$). Consider the operation of composition of functions.

Prove that this is a *model* for the axiomatic system that describes group structures. That is, prove that $G$ is a group. For this you need to check the four axioms. In this case it will be convenient to check them in a different order than the way they are stated in the definition in the book (but of course it doesn't matter in the end which order you check them, as long as you know that they are all true).

i. First show that composition of functions is associative: given three functions $f, g, h$, show that the compositions $f \circ (g \circ h)$ and $(f \circ g) \circ h$ are equal as functions on $\mathbb{R}$.

ii. Next say which element should be the identity object of $G$, and prove that it satisfies the identity axiom.

iii. Now show that each element has an inverse.

iv. Finally, prove that A1 is satisfied: show that if $f$ and $f'$ are elements of $G$, then so is $f \circ f'$.

**Exercise 2.**

a. *[5 points]* Exercise 1.4.6.

b. *[5 points]* Exercise 1.4.7.

c. *[5 points]* Exercise 1.4.8: prove that the identity element of a group $G$ is unique. That is, suppose that $G$ contains an element $e'$ such that for every element $x \in G$, $x \circ e' = x$. Show that $e' = e$, using only the four axioms and the previous two exercises.

**Exercise 3.** Look at the axioms for Four Point Geometry given on page 32 of the book.

a. *[5 points]* Exercise 1.5.4.

b. *[5 points]* Exercise 1.5.5.

c. *[5 points]* Exercise 1.5.6.

d. *[5 points]* Exercise 1.5.7.

*Remember that in addition to the points assigned to each question, you will receive up to five further points for neatness and organization.*