Last time: iterated integrals

Let $D = [0, 2] \times [-3, 1]$. Find $\iint (3x^2 + 3y^2) dA$.

(a) -12

(b) 42

(c) 88

(d) Some other number

(e) I don't know

(If you're done, try integrating using the opposite order of integration to what you used the first time. You should get the same answer.)

Recall: Fubini's Theorem

Theorem

Let f be a continuous function on $D = [a, b] \times [c, d]$. Then

$$\iint_D f(x, y)dA = \int_c^d \int_a^b f(x, y)dxdy = \int_a^b \int_c^d f(x, y)dydx.$$

More generally, this is true if f is bounded and is continous except at a finite number of smooth curves, provided that the iterated integrals exist.

Regions of Type I

We say $D \subset \mathbb{R}^2$ is of Type I if it is of the form

$$D = \{(x, y) \mid a \leq x \leq b \text{ and } g_1(x) \leq y \leq g_2(y)\},$$

where $g_1, g_2 : [a, b] \to \mathbb{R}$ are continous functions.

Theorem

Let $f(x, y)$ be a continous function on a region D of type I as above. Then

$$\iint_D f(x, y) dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy dx.$$

Practice with regions of type II

Recall the region D enclosed by the lines $x = 0$, $y = 1$, and the curve $y = x^2$.

To show that D is a region of type II, we need to find numbers c and d and continuous functions h_1, h_2 on the interval $[c, d]$ such that

$$D = \{(x, y) \mid a \leq x \leq b \text{ and } g_1(x) \leq y \leq g_2(y)\}.$$

(a) I don't know what to do.

(b) I'm working on it.

(c) I have answers, but they don't match with my neighbour's.

(d) We agree.

Integrating over a region of type II

Let $D = \{(x, y) \mid 0 \leq y \leq 1 \text{ and } \sqrt{y} \leq x \leq 1\}$. How would you find the area of D? Fill in the blanks in the following formula:

$$\text{Area of } D = \int_{?}^{?} \int_{?}^{?} ?\ d?\ d?.$$

(a) I don't know what to do.

(b) I'm working on it.

(c) I have answers, but they don't match with my neighbour's.

(d) We agree.

Practice with integrating over polar rectangles

Let $D = \{(x, y) \mid 1 \leq x^2 + y^2 \leq 4 \text{ and } 0 \leq y\}$ as before. What is

$$\iint_D y dA?$$

(a) 0
(b) $\frac{14}{3}$
(c) 3
(d) $3\pi y^2$
(e) I don't know.