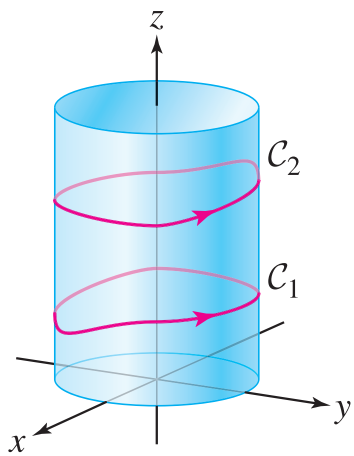
Thursday, April 25 ∗∗ *More on Stokes' Theorem*

1. Let $\mathbf{F} = \langle y^2,\ x^2,\ z^2\rangle$. Show that

$$\int_{C_1} \mathbf{F}\cdot d\mathbf{r} = \int_{C_2} \mathbf{F}\cdot d\mathbf{r}$$

for any two closed curves as shown lying on a cylinder about the z-axis.



2. Consider the surface T which is the intersection of the plane $x+2y+3z=1$ with the first octant.

   (a) Draw a picture of T.

   (b) Use Stokes' Theorem to evaluate $\int_{\partial T} \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F} = \langle y,\ -2z,\ 4x\rangle$. Here, you should orient ∂T counterclockwise when viewed from $(2,2,2)$.

3. Carefully explain how Green's Theorem is actually a special case of Stokes' Theorem.

4. Work the following problem.

20. The magnetic field $\mathbf{B}$ due to a small current loop (which we place at the origin) is called a **magnetic dipole** (Figure 18). Let $\rho = (x^2 + y^2 + z^2)^{1/2}$. For ρ large, $\mathbf{B} = \text{curl}(\mathbf{A})$, where

$$\mathbf{A} = \left\langle -\frac{y}{\rho^3}, \frac{x}{\rho^3}, 0 \right\rangle$$

(a) Let $\mathcal{C}$ be a horizontal circle of radius R with center $(0, 0, c)$, where c is large. Show that $\mathbf{A}$ is tangent to $\mathcal{C}$.

(b) Use Stokes' Theorem to calculate the flux of $\mathbf{B}$ through $\mathcal{C}$.

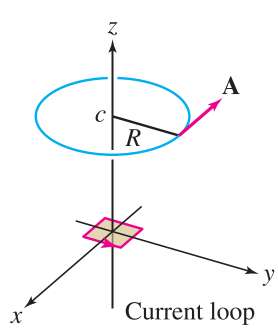


FIGURE 18