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Preface to the Focus Issue: Chaos Detection Methods and Predictability

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This Focus Issue presents a collection of papers originating from the workshop *Methods of Chaos Detection and Predictability: Theory and Applications* held at the Max Planck Institute for the Physics of Complex Systems in Dresden, June 17–21, 2013. The main aim of this interdisciplinary workshop was to review comprehensively the theory and numerical implementation of the existing methods of chaos detection and predictability, as well as to report recent applications of these techniques to different scientific fields. The collection of twelve papers in this Focus Issue represents the wide range of applications, spanning mathematics, physics, astronomy, particle accelerator physics, meteorology and medical research. This Preface surveys the papers of this Issue. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4884603]

Being able to distinguish chaoticity from regularity in deterministic dynamical systems, as well as to determine the subspace of the phase space in which instabilities are expected to occur, is of utmost importance in areas as disparate as astronomy, particle physics, and climate dynamics. Is the solar system stable? Are accelerator beams consisting of elementary particles stable? How can we improve the behavior of these accelerators by avoiding chaotic zones? How shall we construct ensembles in numerical weather prediction or climate dynamics in order to capture most of the dynamics on a given time scale and to increase predictability? These are some of the questions that are studied in the papers of this Focus Issue using methods from dynamical systems theory, symbolic dynamics and network theory.

I. THE PAPERS IN THIS ISSUE

Recent times have seen a plethora of methods which are able to distinguish regular from chaotic dynamics. Several papers utilize some of these methods to study fundamental aspects of dynamical systems as well as dynamical aspects of particular systems. Mulansky[8] is concerned with the connection between microscopic dynamics and macroscopic diffusion. Considering a chain of nonlinear oscillators, the author uses Lyapunov exponents to investigate the phase space structure and the probability that a random initial condition experiences chaotic dynamics. The author finds different microscopic scaling regimes and relates them to macroscopic diffusive quantities such as wave packet spreading. Antonopoulos *et al.*[1] also study the diffusive behavior of dynamical systems. They consider a disordered Klein-Gordon chain and investigate the break down of the central limit theorem in weakly chaotic regimes. They are able to match the statistical properties of partial sums of the position variables with non-Gaussian q-statistics. Gottwald and Melbourne[5] propose a conjecture on the nature of attractors of smooth dynamical systems, which claims that typical dynamical systems are either regular or strongly chaotic, exhibiting good statistical properties such as the central limit theorem. The authors devise a numerical test for verifying the conjecture.

Chaotic indicators are shown to reveal interesting structures of dynamical systems. Kyriakopoulos *et al.*[6] study the vortex dynamics of a Bose-Einstein condensate. Using the Smaller ALignment Index (SALI), they determine the chaotic fraction of the phase space and its dependence on physically important quantities of this system such as the energy and angular momentum. Barrio, Blesa, and Serrano[2] combine the chaos indicator OFLI2 (Orthogonal Fast Lyapunov Indicator) in conjunction with bifurcation analysis software to unveil the mechanisms of the transition to unbounded dynamics in the dissipative Rössler system. Papaphilippou[9] addresses the problem of particle beams in real accelerators whose dynamics is highly nonlinear, mainly due to the presence of electromagnetic fields used for guiding and focusing these beams. The author describes how frequency map analysis can be used to detect the chaotic regions and the higher-order resonances preventing stability of particle beams. This provides constructive means to design and optimize particle accelerators.

A beautiful visualization technique is introduced by Lange *et al.*[7] Employing an iterative contraction method, they unravel the global organization of regular tori in four-dimensional (4D) symplectic maps. Constructing 3D phase space slices, they show how tori are organized around a skeleton of elliptic 1-tori.

Network analysis has gained much interest recently in studying time series and analyzing dynamical regimes. Sun *et al.*[12] construct weighted and directed networks from a given time series using symbolic dynamics to define nodes and connections. This is an interesting alternative to phase space partitions avoiding some of the drawbacks of

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traditional embedding procedures. Analyzing topological network structures is shown to reveal dynamical transitions of the underlying dynamical system. Charakopoulos *et al*.[4] investigate experimental temperature observations of a turbulent heated jet by constructing a network. The topological characteristics of the network are shown to reflect different dynamical regimes of the jet. Symbolic dynamics is used directly by Ravelo-García *et al*.[11] in conjunction with linear regression models to study heart rate variability and promises to be a low cost alternative for sleep apnea screening.

Predicting the behavior of complex dynamical systems given some (generally noisy) observations is of utmost importance. In the context of the atmospheric and oceanographic sciences, the process by which the current state of a system, as well as the uncertainty associated with the analysis, is determined is called *data assimilation*. This procedure is made difficult by the fact that not all variables can be observed nor does one have access to all parameters of a given forecasting model. Furthermore, possible underlying chaotic dynamics imply sensitive dependence on initial conditions, spoiling model forecasts. Parlitz, Schumann-Bischoff, and Luther[10] address the problem of *observability* in nonlinear chaotic dynamical systems, i.e., the problem that given a certain observation one may not be able to determine all state variables and/or unknown parameters of the model. Using delay coordinates, the authors investigate which parameters and state variables can be determined for a given observation. Bellsky, Kostelich, and Mahalov[3] consider the problem of targeted observations and ask the question which variables should be observed to improve parameter estimation in data assimilation. They consider the attractive framework of ensemble filters whereby an ensemble of initial conditions is propagated rather than a single trajectory providing a Monte-Carlo estimate of the error covariance of the model forecast. Targeted observations are identified as those variables for which the forecast error covariance is largest.

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